Kirill Levine

BASICS OF ELECTROSTATICS

TUTORIAL WITH PROBLEMS AND EXAMPLES

Additional reading

Science Impact

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Basics Of Electrostatics. Tutorial with problems and examples

This book covers basics of electrostatics. Can be used as material for lectures and training classes for graduate and with little surgery for undergraduate students. Algebra based with the elements of calculus.

Pre-requisites: basic mechanics, thermodynamics, algebra.

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Introduction

This textbook contains theoretical background and problem solving examples in electrostatics, which is supposed to be useful for students for self-studying and as an additional reading.

Beginning from historical aspects are reviewed experiments on generating electrical charges and their interaction (Coulomb's law), introduced quantities of electric filed, electric potential. Their relationship is discussed. Introduced quantity of flux. Gauss theorem is given in integral and differential form. Necessary mathematical definitions and formalization with Hamilton operator is given. Visualization of electric field with electric field lines and equipotential surfaces is provided.

Each chapter ends with example of problem solutions.

Problems for solving are given at the end.

SUBJECT OF ELECTROSTATICS

This is not exactly clear which experiments have led mankind to the idea of the existence of electric charges. Maybe this was experiments with amber, which has the ability to electrify. This experiment can be easy repeated at home. If you rub a piece of amber with wool, small dielectric objects, for example scraps of paper, will be attracted to it.

Phenomenon of electrifying can be observed while wearing clothes which includes synthetics. When wearing such clothes, sparks might occur between body and surrounding objects.

Those phenomenon's by themselves cannot yet be the evidence for the existence in nature of electric charges, rather, they can lead to such thoughts.

Without knowing laws of electrostatics, students usually hypothesize regarding amber experiment: «upon friction of amber with wool, electrons move from one piece to another. Opposite charges attract. And this is the reason why, due to Coulomb's law, amber (or ebonite stick, or glass stick if to conduct experiment with them) would attract scraps of paper». However, electrostatics is not limited to Coulomb's law, and hypothesis settled by students fails logical testing. If charges are transferred between amber and wool, why scraps of paper, which are uncharged, attract to amber? Charges are not likely to move from wool to paper, as both of those materials are insulators (possess such a high electrical resistance that virtually no electrical conductivity occurs through them). If so, why amber attracts electrically uncharged scraps of paper? Consequently, provided hypothesis is not correct.

Correct explanation of amber experiment will be given further in this book.

Electrostatics is a branch of physics that deals with stationary *(time independent) electric charges and fields, created by charges at rest.*

INTERACTION OF STATIC ELECTRIC CHARGES

Experimental evidence of the existence of electric charges follows from Coulomb's law, while their discrete nature was shown in Millikan's experiment [1]. In order to conduct Coulomb's experiment, it is necessary to separate electric charges. One of the ways to do it is to make friction between amber and wool (and this way it was done historically). In scholar laboratories popular device for this purpose is electrophori machine.

CHARGE SEPARATION EXPERIMENT

Electrophori machine (EM) (Fig. 1) as charge separation system was popular until late XVIII century. In EM dielectric discs are rotating in opposite directions. Discs carry metallic bars, discharged by metallic brushes. Metal strips are deposited on the discs. The charge from rotating discs is removed by static brushes. The design of the machine allows removing charges in such a way that the positive and negative charges are accumulated on different plates of the capacitor, which in our case is the so-called Leyden jar.

Leyden jar *is a glass cylinder, one of the walls of which is wrapped in a sheet of metal foil.* The system consisting of two Leyden jars is a conventional electric capacitor, in which charge of one sign accumulates on one, and opposite - on the other plate, separated by a dielectric. In this example, the mechanical energy is converted into electrical energy. An explanation of the work of the EM phenomenon is described on page 42, after phenomenon's on which it is based are explained.

If one connects the wires to the different poles of the Leyden jar and pulls them together, the charges will begin to transfer from one wire to the other. Such a process in the air is accompanied by the formation of a spark and a characteristic crackle (Fig. *1*).

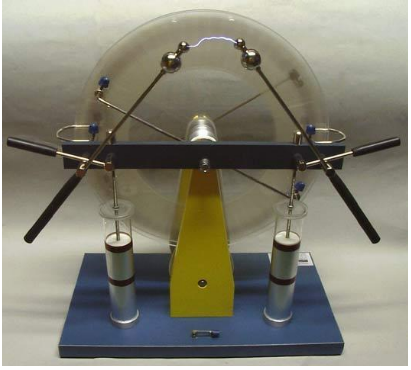


Fig. 1. Electrophori machine.

Electric charges, obtained with the help of EM, can be used to set the Coulomb's experiment.

COULOMB'S EXPERIMENT

Consider two electrically conducting balls. One of them is immovably fixed, the other is balanced by an elastic filament so that it is in a balanced state in contact with the first (Fig. 2). We will consider the balls to be absolutely identical. Let's give one of the balls an electric charge, for example, taken from the Leiden jar. The charges are distributed equally between them. For example, if the first 1000 electrons passed, then after contact on both will be 500. And then we will observe what Coulomb observed. The balls of the same name will start to repel. The strength with which they repel is easy to find, knowing the rotational elasticity of the filament. It turns out that the force with which the balls repel is inversely proportional to the square of the distance between the centers of the balls. We will modify the experiment. Considering that the charges on the Leiden jars are exactly equal and opposite in sign, we will transfer to balls opposite charges from different Leiden jars. Preliminarily we will ensure that the equilibrium position corresponds to a certain angular distance between the balls. Then the filament on which the second ball hangs is twisted to some angle, indicating the attraction of the balls. The magnitude of attraction force will be inversely proportional to the square of the distance between their centers, just as in the first case.

Let's think about the charge carriers in the first and second of the considered cases. The carrier of a negative charge is an electron. The concept of "electron" as an indivisible particle was introduced by the electrochemist J. Stoney in 1894, he was discovered later: in 1897, by E. Wiechert and J. Thomson. In our environment there is no such a "convenient" particle, like an electron, to carry a positive charge. Positive charge is carried by "proton", which is three orders heavier than electron. Nevertheless, current equally easily flows as in negative, as in positive direction. This is because flow of negative charges in one direction (for example, from left to right) is the same as flowing of positive charges (protons) in the opposite direction (from right to left).

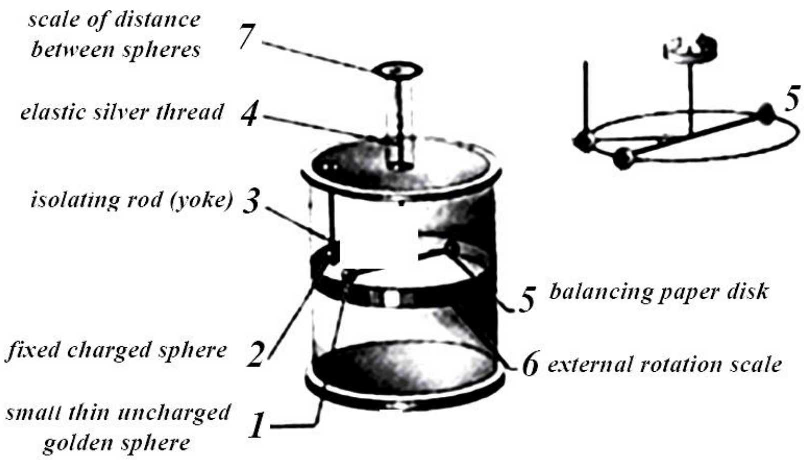


Fig. 2. Schematics of Coulomb’s experiment.

Concluding, **Coulomb’s law** can be formulated as: *interacting force between two point charges in vacuum is inversely proportional to the square of the distance between them. It is directed along the straight line connecting the charges. It is the force of attraction, if the signs of charges are different, and the force of repulsion, if these signs are the same.* (Fig. 3).

In this form the law was formulated by Charles Coulomb in 1785.

In scalar form (simplistically), the law can be written as

$$F = k\frac{|q_1|\cdot|q_2|}{r^2} \tag{1}$$

where r – distance between charges (Fig. 3).

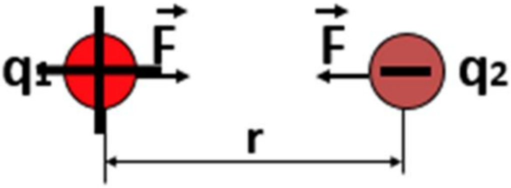


Fig. 3 Illustration to Coulomb's law.

This is more correct to formulate Coulomb's law in vector form:

$$\vec{F} = k\frac{|q_1|\cdot|q_2|}{r^2}\frac{\vec{r}}{|\vec{r}|} \tag{2}$$

where q_1 and q_2 charges, $\vec{r}_1$ and $\vec{r}_2$ radius-vectors drawn from the origin to interacting charges. This has to be considered that (Fig. 4):

$$\vec{r} \equiv \vec{r}_{21} = \vec{r}_2 - \vec{r}_1 \tag{3}$$

and

$$r \equiv |\vec{r}| = \sqrt{\vec{r}^{\,2}} \tag{4}$$

.

Symbol «≡» has meaning of «re-assigning» or, by other words, «same identity».

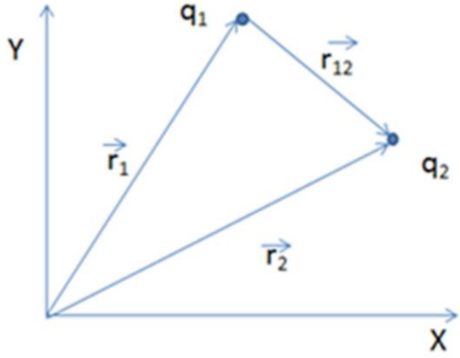


Fig. 4. Vector subtraction law reminder.

Sometimes Coulomb's law is formulated as

$$\vec{F}_{21} = k \frac{|q_1| \cdot |q_2|}{\left|\overrightarrow{r_{21}}\right|^3} \overrightarrow{r_{21}} \quad , \tag{5}$$

Where (5) is essentially (2) in slightly different form. Coefficient k (Coulomb's constant) is

$$k = \frac{1}{4\pi\varepsilon_0} \tag{6}$$

and

$$\varepsilon_0 = 8{,}85 \cdot 10^{-12} \text{ F/m}, \tag{7}$$

spelling “epsilon naught” - electric constant or vacuum permittivity. Both k and ε_0 are met only in SI metrical system and are useful only for converting engineering quantities. They do not possess fundamental physical sense and are absent in SGS system, which is more frequently used by physics rather than engineers. Unit «Coulomb» (C), in (*7*) is a unit of electrical charge and is equal to charge transferred by current of 1 ampere (A) in 1 second.

COULOMB’S LAW – ACTION AT DISTANCE LAW

Coulomb's law is the so-called "action at distance" law, which is carried out through “long range forces” since the force of the interaction of charges is proportional to the square of the distance in the minus two degree. The same dependence has a gravitational interaction, which is known to act between stars within one galaxy and galaxy in clusters and hold them together. However, at long distances, no one checked it, so the "action at distance" property of the Coulomb’s law is a mathematically justified hypothesis. We note that forces proportional to higher degrees of distance in the denominator, such as Van der Waals forces (Table 3), unlike “long range forces”, are called “short-range forces”.

ELECTRIC CHARGE PROPERTIES

Coulomb’s law is based on the experimentally established fact: electrified objects interact (attract/repel) to/from each other. In order to characterize the force of interaction, the concept of charge is introduced, and in order to explain the very fact of interaction, the concept of an electric field (EF) is introduced. Charge is a characteristic of matter, while EF is a form of the existence of matter. About the electric field can only be said that it manifests itself in the interaction of charges, and this is the form of existence of matter. Without Coulomb's law, there would be nothing known about the electric field.

The most important properties of electric charge are:

1. Discreteness.
2. The conservation of electric charge.
3. Invariance with respect to the *Lorentz transformations*.

Transformations of a frame of reference due to relativistic effects. See Einstein's general theory of relativity [2].

The property of discreteness lies in the fact that the smallest portion of experimentally measured electric charge is equal to the charge of an electron. From nuclear physics it is known that there are also fractional charges equal to one-third and two-thirds of the electron charge. Particles bearing such charges are called quarks. Protons and neutrons consist of quarks. However, quarks were not detected in the free state. The absence of free quarks in nature is called the *quark confinement principle*. There is still the possibility of availability of free quarks left after the Big Bang, but finding them, if they really exist, is very difficult.

The most important charge carriers for us, besides electrons, are protons (Table 1) whose charge is exactly equal to the electronic charge, but opposite in sign. It is also useful to distinguish anti-particles. Examples of them are positrons – particles with a charge that coincides with the charge of an electron, but opposite in sign, and a mass exactly equal to the mass of the electron, and antiprotons – particles with a charge equal to the charge of the electron (i.e., a negative charge) and a mass equal to mass of the proton. To complete the picture, should be mentioned particles, neutrons, whose mass is close to the mass of the proton, but the charge is zero. Unlike previously described particles, neutrons are not stable. The average time of their life is about 18 minutes, which is neither long nor short from the point of view of nuclear physics.

The stability of positrons, electrons, protons and antiprotons are stable. Their lifetime is infinitely long. Proton and antiproton, electron and positron are considered anti-particles to each other. If particle meets inti-particle, they both convert to something else. *Reaction of particle and anti-*

particle is called **annihilation.** It usually results in transferring matter to electromagnetic waves in accordance with Einstein's equation

$$E=mc^2 \quad (8)$$

Where m is mass in kilograms, c is speed of light in m/s and E is energy in joules. The frequency of electromagnetic waves ν is defined by Plank's formula:

$$E=h\nu \quad (9)$$

Where h is a Planks constant equal to $6{,}626\ 070\ 040\cdot10^{-34}$ J/s.

Table 1. Some elementary particles and their main properties.

Name/Characteristics	Proton	Neutron	Electron
Charge	$+1.6\cdot10^{-19}$ C	0	$-1.6\cdot10^{-19}$ C
Mass	$1.67\cdot10^{-27}$ kg	$1.67\cdot10^{-27}$ kg	$9.1\cdot10^{-31}$ kg
Name/Characteristics	Anti-proton		Positron
Charge	$-1.6\cdot10^{-19}$ C		$+1.6\cdot10^{-19}$ C
Mass	$1.67\cdot10^{-27}$ kg		$9.1\cdot10^{-31}$ kg

Strictly speaking, those electromagnetic waves would be gamma quants.

This is an interesting question if our Universe is constructed only from matter (not anti-matter). The explanation is as follows. This is believed that immediately after Big Bang matter and anti-matter was present in the same amount within the precision of a fluctuation. Than the matter entirely reacted with anti-matter, however, the fluctuation left unreacted. Our Universe is constructed from this fluctuation.

For us, only electrons and protons matter in this course. It is theoretically possible to obtain atoms consisting of negatively charged anti-nuclei and positrons located in quantum orbits around them. A substance made from such atoms is called antimatter. If an atom consisting of a proton and an electron is called hydrogen, then the atom, consisting of an antiproton and a positron, is called antihydrogen. Antihydrogen in faint quantities can be obtained in accelerators.

The law of conservation of electric charge (LCEC) is same fundamental as other fundamental laws: conservation of momentum, angular momentum and energy. It has no exceptions. Strictly speaking, the reason why charge is called charge, is because the conservation principle is applicable to it.

An interesting fact about electric charge would be **the principle of electrically neutral Universe**: *the number of positive and negative charges is exactly the same* (Fig. 5), (*10*).

This identity is averaged. In other words, electrical charges of one or the other sign (10) can predominate at specific points of space (11). Mathematically LCEC can be written as

$$\sum q_i^+ + \sum q_j^- = 0 \qquad (10)$$

(globally) or:

$$\sum q_k = const \tag{11}$$

(locally).

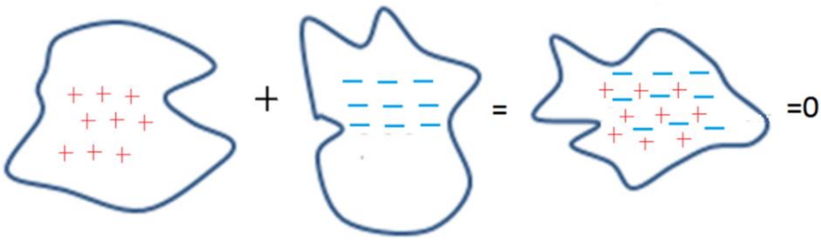


Fig. 5. Law of electrical charge conservation.

To illustrate LCEC, the best way would be to discuss its implementation in chemistry and nuclear physics.

LCEC EXAMPLE IN CHEMISTRY

Consider sodium chloride dissolution in water:

$$NaCl \rightarrow Na^{+} + Cl^{-}$$

In the solid state, the cubic sublattice of sodium ions is positioned within the cubic sublattice of the chlorine ions. When passing into solution, the ions of both signs begin to move chaotically, but the electrical neutrality of the solution as a whole remains in strict accordance with the LCEC.

LCEC EXAMPLE IN NUCLEAR PHYSICS

Consider electron – positron annihilation:

$$e^{+} + e^{-} \rightarrow 2\,h\nu, \qquad (12)$$

where h – Plank's constant, ν – frequency. (12) suggests only one of the possible (but most probable) channels for annihilation of an electron-positron pair, which results in a pair of virtual photons capable of giving birth to a pair of any particles, but more often these "any" particles turn out to be real photons. The total charge of particles on the left-hand side of the reaction is zero. As is known, photons are charge-free particles, so the total charge of particles on the right-hand side is also zero.

> Why number "two" is used to indicate the number of photons. Can only one photon occur?

As for the third fundamental property of electric charge invariance with respect to Lorentz transformations, it means that the charge does not undergo relativistic corrections when going to relativistic velocities.

ELECTRIC FIELD

The Coulomb's law provides the evidence of the existence of a special form of matter - the electric field (EF), which is introduced to explain interaction between charges. The electric field is a vector quantity and is usually denoted by a Latin letter "$\vec{E}$". The dimension of the electric field in SI system is volts per meter (V/m). EF not changing with time is called "electrostatic field".

The most important properties of EF are:

1) Obeys the principle of superposition.

2) Acts on charges in accordance with Coulomb's law.

THE PRINCIPLE OF SUPERPOSITION

Total field can be calculated as a vector sum of separate fields created by composing it charges:

$$\vec{E}_{\Sigma} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n = \sum_1^n E_i \quad , \tag{13}$$

where E_i - field, created by each charge, composing system of n charges (Fig. 6).

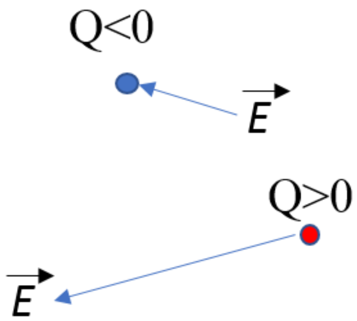


Fig. 6 . Graphical interpretation of EF strength with respect to positive and negative charges.

It is shown graphically for system composed of two charges in Fig. 7.

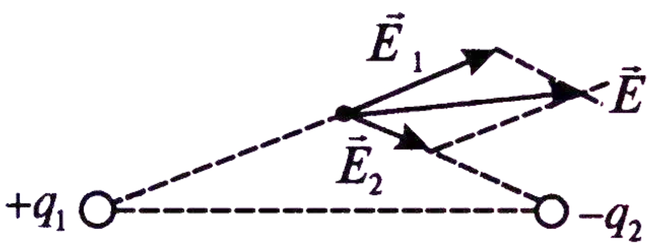


Fig. 7. The principle of electric field superposition.

CHARGE-FIELD INTERACTION

Assuming that force acting between charges is proportional to EF strength, it can be written:

$$\vec{F} = \vec{E} \cdot q \tag{14}$$

Re-writing this formula through Coulomb's law, EF strength can be defined as

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\frac{\vec{r}}{|\vec{r}|} \quad , \tag{15}$$

where $\vec{r} = \vec{r}_2 - \vec{r}_1$ – vector, connecting charges, $\frac{\vec{r}}{|\vec{r}|}$ – unit vector, defining direction from charge q to q'.

Therefore, EF strength is inversely proportional to squared distance (**Ошибка! Источник ссылки не найден.**). In any point in space, the EF is directed along the ray that connects this point of space with the charge. The direction of EF vector is considered to coincide with the direction of movement of the test positive charge placed at the point where there is an EF.

ELECTRIC FIELD AS A CONSERVATIVE FIELD

In the previous topic it was shown that the interaction between the resting charges is carried out through EF by a vector of EF strength. It can be shown that EF is conservative, and the field is a so-called "potential field".

Consider field, created by static (not moving in the frame of reference) charge q. It certainly can move in another frame of reference. But it is taken "laboratory" frame of reference, where it is not moving. For example, in a laboratory, charge created at fixed Leyden jar is immovable. At any place at a probe charge q' acts force $\vec{F}$, which can be represented as

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}\frac{\vec{r}}{|\vec{r}|} = F(r)\frac{\vec{r}}{|\vec{r}|} \quad (16)$$

Where $F(r)$ – force modulus.

In order to proove that EF is a potential field, it needs to be prooved that EF forces are conservative.

Same as with arrows, vector quantities are frequently marked in bold.

From mechanics this is known that any static field of central force is conservative. By other words the work of the forces of this field does not depend on the shape of the path, but only on the position of the final and initial points. EF is central, therefore, it is conservative. This can be confirmed mathematically: by calculating the work done by the electrostatic field created by the charge q in the displacement of the charge q' from point 1 to point 2 (Fig. 8). Movement along the trajectory ($d\boldsymbol{l}$) can be decomposed into two components: along the line connecting the charges ($d\boldsymbol{l}_{||}$), and perpendicular to ($d\boldsymbol{l}_{\perp}$). The work dA will be:

$$dA = \boldsymbol{F}\cdot d\boldsymbol{l} = \mathbf{F}\cdot(d\boldsymbol{l}_{||} + d\boldsymbol{l}_{\perp}) \quad (17)$$

Considering $\boldsymbol{l}_{||} || \boldsymbol{r} \Rightarrow \boldsymbol{F}\cdot d\boldsymbol{l}_{||} = Fdl_{||}cos0^{\circ} = Fdr,$

where

$$r = |\vec{r}|$$

and

$$\boldsymbol{l}_{\perp} \perp \boldsymbol{r} \Rightarrow \mathbf{F}\cdot d\boldsymbol{l}_{\perp} = Fdl_{\perp}cos90^{\circ} = 0$$

Therefore:

$$\mathrm{d}A = \frac{qq'}{4\pi\varepsilon_0 r^2}\mathrm{d}r \qquad (18)$$

And total work of transferring charge along 2 to 1 path is:

$$A_{12} = \frac{qq'}{4\pi\varepsilon_0}\int_{r_1}^{r_2}\frac{dr}{r^2} = \frac{qq'}{4\pi\varepsilon_0}(-\frac{1}{r})\Big|_{r_2}^{r_2} = \frac{qq'}{4\pi\varepsilon_0}(\frac{1}{r_1}-\frac{1}{r_2}) \qquad (19)$$

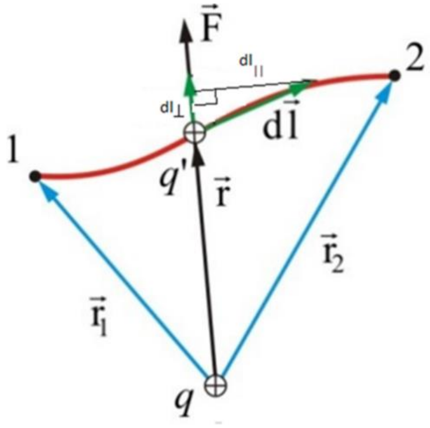


Fig. 8. To the conservancy of electric field.

This equation shows that the work of electrostatic forces does not depend on the shape of the path, but only on the coordinates of the initial and final points of the displacement, therefore it is proved that EF is conservative.

EXAMPLES

CALCULATING WORK OF TRANSFERRING CHARGE

Problem A

Let us determine the work on charge transfer for the following case: the charge of 1 nC is transferred from infinity to a point 0.1 m away from the surface of a metal sphere with a radius of 0.1 m, charged with a surface density 10^{-5} C/m^2.

EF creates force acting on charge. Therefore, when the charge moves in EF, work is performed.

The electric field is conservative. The work can be represented by the difference in the potential energies of a particle in an electric field:

$$A_{field} = W_2 - W_1 = \varphi_2 q - \varphi_1 q = q(\varphi_2 - \varphi_1)$$
$$[A_{field}] = J;$$

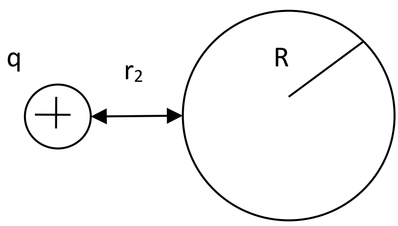
The surface charge density σ is the ratio of the charge of the Q plane to its area S.

$$\sigma = \frac{Q}{S}$$

The surface charge density is a finite quantity characterizing the of charge of an infinite plane.

Given:

$q = 10^{-9}$ C
R = 0.1 m
σ = 10^{-5} C/m^2
$r_2 = 0.1$ m
$r_1 = \infty$



Solution:

$$A = W_2 - W_1;$$
$$W = \varphi \cdot q; \quad \varphi_1 = 0$$
$$A = q(\varphi_2 - \varphi_1) = q\varphi_2 = \frac{1}{4\pi\varepsilon\varepsilon_0} \cdot \frac{Q}{r+R};$$
$$Q = \sigma \cdot S = 4\pi R^2 \cdot \sigma;$$
$$A = \frac{4\pi R^2 \cdot \sigma}{4\pi\varepsilon\varepsilon_0 (r+R)} = \frac{R^2 \cdot \sigma}{\varepsilon\varepsilon_0 (r+R)};$$
$$A = \frac{10^{-5} \cdot 10^{-2}}{8{,}85 \cdot 10^{-12} \cdot 0{,}2} = 56497 \ J$$

Answer: A=57 kJ.

Problem B.

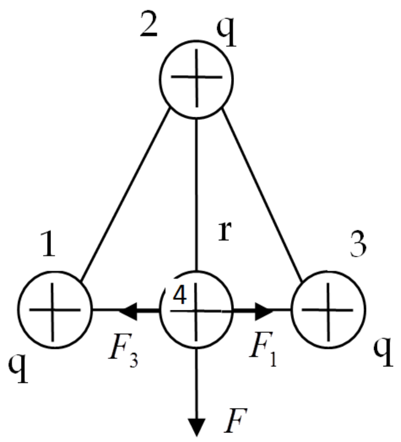
Charges of 1 nC are placed at the vertices of an equilateral triangle with a side of 0.2 m. Equal force acting on the fourth charge, placed in the middle of one of the sides of the triangle, is 0.6 μN. Determine this charge and the field strength at its location.

Given:

q=1 nC

a=0.2 m

F=0.6 μN



Solution:

It can be seen from the figure that the resultant of the charges (1) and (3) on the fourth charge is zero. Consequently, the force acting on this charge is determined by the action of the charge (1) and can be found by Coulomb's law, the distance between the charges is found from a right triangle..

$$F = \frac{q \cdot q_0}{4\pi \cdot \varepsilon_0 r^2};$$

$$r = a \cdot \cos\frac{\pi}{6};$$

Denoting the test charge through q_0, next can be calculated its value and units checked for correct dimensions.

$$F = \frac{q \cdot q_0 \cdot 4}{4\pi \cdot \varepsilon_0 a^2 \cdot 3} = \frac{q \cdot q_0}{3\pi \cdot \varepsilon_0 a^2};$$

$$q_0 = \frac{3\pi \cdot \varepsilon_0 a^2 \cdot F}{q};$$

$$q_0 = \frac{C^2 m^2 N}{N m^2 C} = C$$

$$q_0 = \frac{3\pi \cdot 8{,}85 \cdot 10^{-12} \cdot 0{,}04 \cdot 6 \cdot 10^{-7}}{10^{-9}} = 2 \cdot 10^{-9}\ C;$$

By the relation between the charge and the force, we find field strength.

$$E = \frac{F}{q_0};$$

$$E = \frac{6 \cdot 10^{-7}}{2 \cdot 10^{-9}} = 300\ N/C$$

POTENTIAL

This is possible to calculate the work performed when the distance between charges is changed from r_2 to r_1 (Fig. 8). As was shown above, the electric field is conservative, and forces created by it on charges are central forces, therefore, mentioned work does not depend on the shape of the trajectory, but only on the distance between charges.

$$A_{21} = \frac{qq'}{4\pi\varepsilon_0}\int_{r_1}^{r_2}\frac{dr}{r^2} = \frac{qq'}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)\Bigg|_{r_1}^{r_2} \tag{20}$$

where q and q' - charges. Therefore:

$$A_{21} = \frac{qq'}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{21}$$

If the charges approach the distance r_1 infinity (Figure 10), the work is equal to

$$A_{21} = \frac{qq'}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\Bigg|_{r_2=\infty} = \frac{qq'}{4\pi\varepsilon_0}\frac{1}{r_1} \tag{22}$$

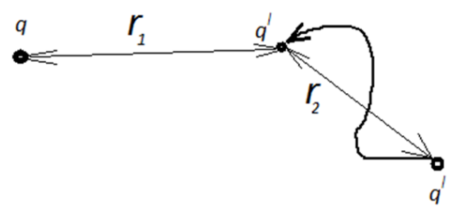


Fig. 9. To evaluation energy of a system of charges

Denoting the distance between the charges as r, and recalling that the work is equal to the change in the potential energy taken with the opposite sign, it can be written:

$$W_{21} = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r} \quad , \qquad (23)$$

where $r = r_2 - r_1$. It is necessary to take into account the sign of charges. This formula is similar to formula for the energy of the gravitational interaction. The absence of a minus sign results from the need to take into account two charge signs. In the case of gravitational interaction, masses are always attracted. The masses do not have different signs. In the case of interaction of charges of the same sign, the energy of the system will be negative.

However, the ratio of energy to charge will be the same for all charges. This quantity is called **potential difference between the two points (*Δφ*),** *which is equal to the ratio of work on the charge transfer between points 2 and 1 to the value of this charge.*

$$\Delta\varphi = \varphi_{21} = \frac{W_{21}}{q} \qquad (24)$$

It can be noticed that physical meaning attributes not to “potential”, but to potential difference. This is agreed to assign the potential of a point removed to infinity as zero. When it is said "potential of some point" – it means the difference of potentials between some reference point, which is at infinity if not specified, and the point of interest.

Thus, the **potential of a point in space created by an electric field** *is numerically equal to the work done by field forces over a unit positive charge when it is removed from a given point to infinity* (or vice versa - it is necessary to do the same job, but with opposite sign, to move a single positive charge from infinity to a given point fields).

The potential is a scalar quantity, which makes it convenient for calculating the EF intensity.

Since the EF is subject to the principle of superposition, the potential obeys additivity. The potential at a point created by a system of charges is equal to the sum of the potentials created by each charge.

$$\varphi_{\Sigma} = \varphi_1 + \varphi_2 + ... + \varphi_n = \sum_1^n \varphi_i \qquad (25)$$

Unit of potential in SI is «volt» (V).

ENERGY OF SYSTEM OF CHARGES

The quantity defined in (23) determines energy of system consisting of two charges. Generalizing the system consisting of several charges: the

energy of system of charges *is equal to the work that must be done in order to bring the charges closer to a given distance from infinity*.

The work calculated in this way is also called the energy of the system of charges. Note that the energy of the system of the same charge is negative (charges tend to move apart), and opposite - positive (attract).

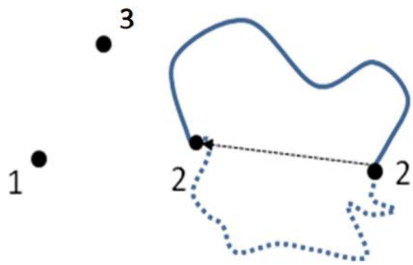


Fig. 10. To charged system energy evaluation.

In similar way the energy of a system consisting of three charges can be found:

$$U = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \qquad (26)$$

Generalizing for an unlimited number of charges:

$$U = \frac{1}{2} kN \sum_{i=1}^{N} \sum_{j=1(i\neq j)}^{N} \frac{q_i \, q_j}{r_{ij}} \tag{27}$$

where N – number of charges. To eliminate the summation of a pair twice, coefficient ½ is used.

EXAMPLE. EVALUATING ENERGY OF CRYSTALLINE LATTICE OF SODIUM CHLORIDE

Consider sodium chloride (NaCl) crystal lattice. This is a volumetric face-centered structure (Fig. 11).

The nature of atom's interaction in it is electrostatic, therefore in order to find internal binding energy of the crystal lattice it is possible to apply (27).

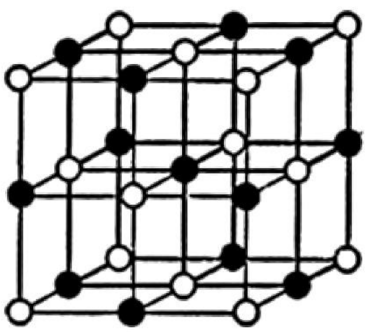


Fig. 11. A three-dimensional face-centered lattice consisting of two types of atoms.

In a face-centered cubic lattice, the sodium sublattice is located within chlorine sublattice. For calculation, will be taken into account the interaction of only the nearest ions (27). Summing over the nearest neighbors taking into account the number of identical pairs:

$$U = \frac{1}{2} N \left[-\frac{6e^2}{a} + \frac{12e^2}{\sqrt{2}a} - \frac{8e^2}{\sqrt{3}a} + \cdots \right] \tag{28}$$

The first term appears from the six nearest sodium ions located at distance a, the second from the twelve chlorine ions located at the corners of the cube, and so on. The more pairs of interactions are considered, the more accurate the result can be obtained. Calculation gives answer:

$$U = -\frac{0{,}8738Ne^2}{a} \quad (29)$$

where a – the shortest distance between atoms of the same sort (lattice constant), N - the number of atoms per unit volume, considering that the number of atoms is twice the number of molecules. Numerical coefficient in formula is also known as Madelung constant.

The determination of the binding energy of a salt crystal by experimental methods yields a value close to the theoretical value, which indicates the correctness of the applied approach.

RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL

Let us take expression between electric field strength $\boldsymbol{E}$, charge q and the work performed by EF to move charge:

$$dA = \boldsymbol{F}d\boldsymbol{l} \quad (30)$$

Recall that work is equal to potential energy change taken with minus sign.

dA=-dU.

Therefore

$$dU = -\boldsymbol{F}d\boldsymbol{l}.$$

Recall

$$\boldsymbol{F}d\boldsymbol{l} = \boldsymbol{E}qd\boldsymbol{l}$$

Than

$$dU = -\boldsymbol{E}q d\boldsymbol{l}$$

From (22) if points 1 and 2 are very close to each other, $\Delta\varphi$ becomes dφ and W_{21} we now denote as dU

$$d\varphi = dU/q \tag{31}$$

than:

$$d\varphi = -\boldsymbol{E}d\boldsymbol{l}\ . \tag{32}$$

Opening the scalar multiplication of vectors:

$$d\varphi = E_x dx + E_y dy + E_z dz \tag{33}$$

Writing projections onto Cartesian axes:

$$E_x = -\frac{d\varphi}{dx} \tag{34}$$

$$E_y = -\frac{d\varphi}{dy} \tag{35}$$

$$E_z = -\frac{d\varphi}{dz} \tag{36}$$

Considering that

$$\vec{E} = \vec{\imath}E_x + \vec{\jmath}E_y + \vec{k}E_z \tag{37}$$

where ***i, j, k*** –unit vectors, it is possible to "construct" ***E*** through its projections on the axes of the Cartesian coordinate system, using Hamilton's operator denoted by Greek letter "nabla" ($\vec{\nabla}$):

$$\vec{\nabla} = \vec{i}\frac{d}{dx} + \vec{j}\frac{d}{dy} + \vec{k}\frac{d}{dz} \quad (38)$$

This operator is convenient because, being formally substituted into different formula's, it produces mathematically correct expressions from the point of view of vector-based calculus. Although the nabla operator is a differential operator, it can be algebraically operated as regular vector.

Substituting (34 - 36) into (37) it is possible to write:

$$\vec{E} = -\left(\vec{i}\,\frac{d\varphi}{dx} + \vec{j}\,\frac{d\varphi}{dy} + \vec{k}\,\frac{d\varphi}{dz}\right) =$$
$$= -\left(\vec{i}\,\frac{d}{dx} + \vec{j}\,\frac{d}{dy} + \vec{k}\,\frac{d}{dz}\right)\varphi \quad (39)$$

As EF is a continuous quantity filling the space, potential is a number that can be assigned to each point of space. It's easy to see that quantity in brackets shows changing of our digital quantity in any direction. In mathematics changing in some direction is called "gradient". It has a mathematical definition that we now omit. This definition does not depend on coordinate system. When we want to underline that it does not depend on our coordinate system, it is possible to write

$$\vec{E} = -grad\varphi \quad (40)$$

Which is the same as

$$\vec{E} = -\vec{\nabla}\varphi \quad (41)$$

Nable operator has different appearance in different coordinate systems, the one above is in Cartesian coordinates.

This is easy to notice that potential contains all the information about the electric field being a scalar quantity, while EF strength is a vector quantity, so in a number of cases electric potential is more convenient quantity to operate with rather than EF.

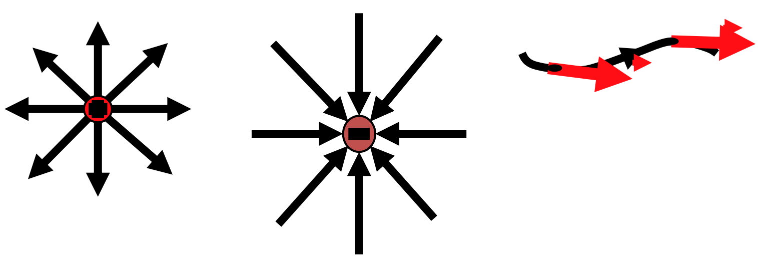
GRAPHICAL REPRESENTATION OF ELECTRIC FIELD

To solve practical problems, it is convenient to represent EF with the help of so-called "electric field lines" and "equipotential surfaces".

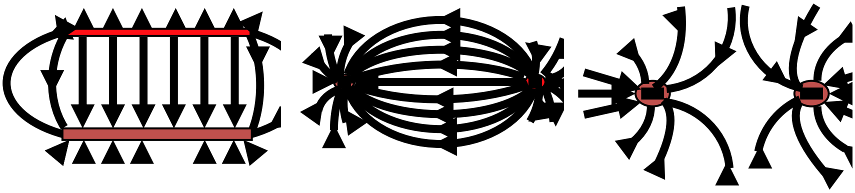
ELECTRIC FIELD LINES

Electric field lines – *Gaussian lines these are continuous lines, tangent to which at each point through which they pass, coincide with the vectors of EF strength.*

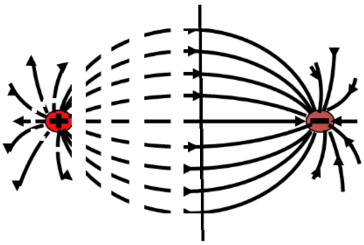
An example of EF lines for charges of various configurations is shown in Fig. 12. As can be seen, the lines near the point charge always diverge radially from the point at which the charge is located (Fig. 12 (A, B, E, F, G)). This is understandable: for $r \to 0$ in Eq. (15), $E \to \infty$, therefore the influence of other EFs sources in the vicinity of the charge can be neglected. Significantly far from the system of charges, EF lines curving increases (Fig. 12 (C)).



A) B) C)



D) E) F)



G)

Fig. 12 – EF lines for cases:
A) point positive charge,
B) point negative charge,
C) an arbitrary system of charges,
D) two oppositely charged planes,
E) two opposite charges
F) two charges of the same sign
G) system consisting of a charge and a conducting plane

For the case of two oppositely charged planes of finite sign, applying the superposition principle, it can be seen that at a sufficient distance from the edges EF lines are parallel, Fig. 12 (D). To construct the image of the lines created by charge and conducting plane, it can be applied the so-called

reflection principle: *charge located near infinite conducting plane interacts with the plane in the same way as if on the other side of the plane exists charge of an equal magnitude, but opposite in sign* (Fig. 12 (G)).

Thus, EF lines always begin on the positive, and end on a negative charge.

EQUIPOTENTIAL SURFACES

For the graphical representation of the potential, this is convenient to use so-called "equipotential (EP) surfaces" - surfaces connecting points of equal potential.

Let us consider an example of constructing an EP surfaces for the case of two conductors under different potentials: a metal plate (1) and a rod (2). Such a system can be assembled as follows. Take a rectangular cuvette filled with an electrically conductive solution, for example a 1% solution of sodium chloride (Fig. 13). In a cuvette there is a plate (1) parallel to one side, and on the opposite side it is vertically placed a rod (2). The battery (4) creates a potential difference between (1) and (2). The potential difference between the probe (3) and the electrode (2) is measured by a voltmeter V.

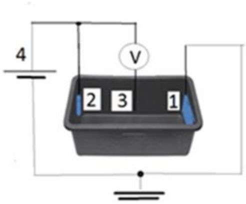


Fig. 13. Experimental setup for measuring electrical potential.

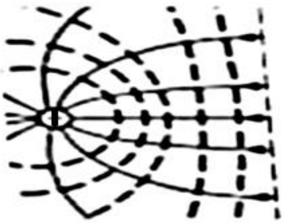


Fig. 14. EP surfaces projection to the plane of a cuvette – EP lines, shown dashed.

To construct equipotential surfaces, it is necessary to plot the diagram of points attaining the same potential inside the rectangle with sides corresponding to cuvette. Looking from the top, equipotential surfaces will look like lines (Fig. 14).

Analyzing the picture, it can be noticed that the potential of points close to a flat metal electrode is approximately the same, being more and more the less with increasing the distance from plane. Consequently, *the potential of all points of the conducting plane is equal*. This is, unconditionally true only a "good" conductor for example, copper at small sizes.

It is easy to see that the *EP surfaces and EF lines are always perpendicular*, which also follows from (40).

DIELECTRIC POLARIZATION

ELECTRICAL DIPOLE

The most important case of an electrically charged system is the case of two electric charges of different sign, but identical in modulus, shown in Fig. 12 (E): electrical dipole.

Electric dipole *is a system consisting of two electrical charges of equal value, but of an opposite sign.*

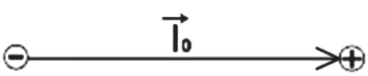


Fig. 15. Electric dipole.

Dipole shoulder *(l_0) is vector connecting the negative charge with the positive one.*

Dipole moment ($\boldsymbol{p}$) *is the vector equal to the product of the dipole arm by the charge modulus, and directed from the negative charge to the positive one [C m].*

$$\vec{p} = q\vec{l}_0 \qquad (42)$$

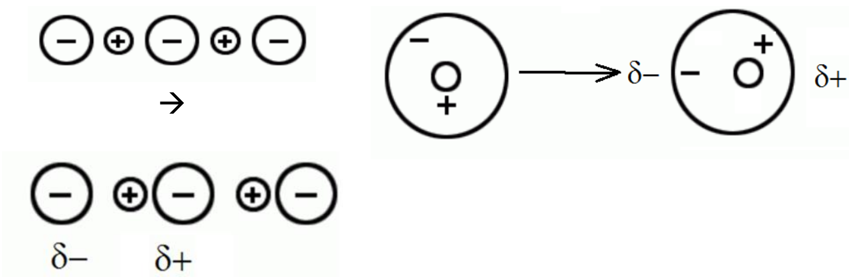
The concept of a dipole is very important because under certain conditions many electrically neutral molecules are able to acquire the properties of a dipole [3].

polarization is *dipole moment acquiring by a substance under the action of external electric field.* It is discussed in detail in the course of "physics of condensed state". A detailed description of polarization theory was made by Debye [4, 5].

Peter Debye, the founder of the theory of dielectric polarization.

The mechanism of polarization can be simplified as follows. Every substance has in its structure an equal number of positive and negative charges. An example is a crystal of table salt, which has an ionic structure (Na^+Cl^-). Another example is hydrogen atom in which a negatively charged electron is spherically symmetrically distributed around a point positively charged nuclei.

When the substance is introduced into the electric field, the charges are shifted slightly relatively to their equilibrium position, forming a structure consisting of electric dipoles (Fig. 16).



Polarization of ionic (polar) dielectric

A

Polarization of non-polar dielectric

B

Fig. 16. Examples of dielectrics polarization under insertion into an external electric field.

Polarization of molecules of matter under the action of an electric field, consisting in the appearance of dipoles in the substance, leads to a weakening of an electric field inside the substance. Quantitatively, this kind of attenuation can be characterized by so-called «dielectric permittivity», in short «permittivity» ε.

Permittivity *is the ratio of the electric field in vacuum to that in the substance.*

As follows from its definition, permittivity of vacuum is equal to one. Since the majority of substances weaken the electric field, ε of them is greater than 1 (Table 2).

Table 2. Permittivity of some substances.

Substance	**ε**
Vacuum	1
Air (normal conditions)	1,000… (slightly higher than 1)
Water	81
Plastics	3 – 7

Example. Calculating electric potential, created by electric dipole

Electrical potential created by a dipole (Fig. 17) can be calculated by summing potentials created by two point charges of opposite charges, applying (23) and (*24*):

$$\varphi_+ + \varphi_- = \frac{q}{4\pi\varepsilon\varepsilon_0 r_+} - \frac{q}{4\pi\varepsilon\varepsilon_0 r_-} = \frac{q}{4\pi\varepsilon\varepsilon_0 r_- r_+} \quad (r_- - r_+)$$

Under fulfilling condition

$$r >> l_0$$

at sufficiently large distance from dipole, it can be taken

$$r_+ r_- \approx r^2;$$

and (Fig. 17).

$$|r_{-} - r_{+}| = l_0 \cos\theta$$

Than:

$$\varphi_{+} + \varphi_{-} = \frac{q}{4\pi\varepsilon\varepsilon_0 r^2} \quad l_0 \cos\theta$$

Or, considering (42), at points located far enough from dipole, potential is equal to

$$\varphi = \frac{p\cos\theta}{4\pi\varepsilon\varepsilon_0 r^2} \qquad (43)$$

which can be conveniently re-written trough scalar product as

$$\varphi = \frac{\vec{p} \cdot \vec{r}}{4\pi\varepsilon\varepsilon_0 r^3} \qquad (44)$$

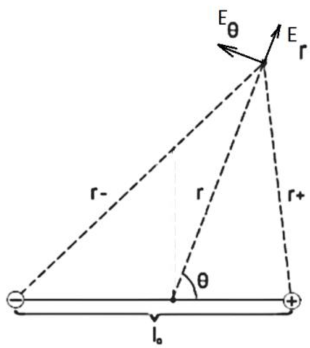


Fig. 17. To electric dipole potential calculation.

Example. Calculating dipole electric field strength

In order to calculate the strength of the electric field produced by dipole, can be used equation (40) applied to (43). It is rational to into account that two charges that constitute dipole, and the point of space in which the intensity is calculated, form a plane. (Three points always define a plane). Therefore, the problem can be solved in polar coordinates by differentiating with respect to distance and angle using the mathematical rules that exist for this.

$$E_r = -\frac{\partial\varphi}{\partial r} = \frac{2p\cos\theta}{4\pi\varepsilon\varepsilon_0 r^3}$$

$$E_\theta = -\frac{1}{r}\frac{\partial\varphi}{\partial\theta} = \frac{p\sin\theta}{4\pi\varepsilon\varepsilon_0 r^3}$$

It should be considered that E_θ component is always perpendicular to E_r (Fig. 17). Therefore, EF strength by modulus is equal to:

$$E = \sqrt{E_r^2 + E_\theta^2} = \frac{1}{4\pi\varepsilon\varepsilon_0}\frac{p}{r^3}\sqrt{1 + 3cos^2\theta}$$

From obtained formulas it can be seeing that on longitudinal dipole axes

$$\theta = 0,\ E_z = 0,\ E = E_r = \frac{2p}{4\pi\varepsilon_0 r^3},$$

while on perpendicular axes

$$\theta = \pi/2,\ E_r = 0,\ E = E_\theta = p/4\pi\varepsilon_0 r^3.$$

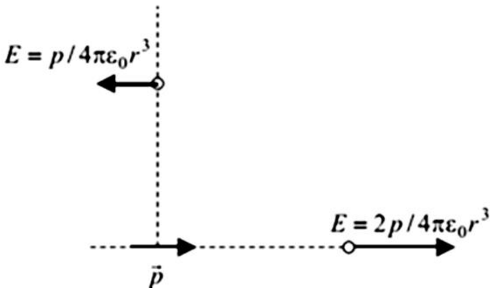


Fig. 18. To dipole electric field strength calculation.

CALCULATING FORCE, ACTING ON DIPOLE IN ELECTRIC FIELD

Two cases can be considered.

UNIFORM FIELD

If the electric field is uniform, then the same forces will act on the positive and negative charges. In the case, when the direction of the field parallel to the dipole moment, the forces completely compensate each other and nothing happens with the dipole (Fig. 19).

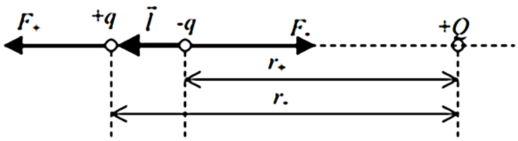


Fig. 19. Dipole in uniform electric field, which direction is parallel to dipole moment.

If the dipole moment is not parallel to the field, then the torque will act on the dipole, tending to turn it in the direction of the field. This torque is equal to the product of the dipole moment by the electric field strength.

$$\vec{N} = [\vec{l}q\vec{E}] = [\vec{p}\vec{E}]$$

NON-UNIFORM FIELD

If the field is nonuniform, then, as in the previous case, there will be a force tending to turn dipole in the direction of the field. In the case when the dipole is already aligned by field, which is essentially inhomogeneous at distances comparable with the dipole arm, different forces will act on the negative and positive charge, the resultant of which will be directed toward EF strength increasing. As an example, assume that the field is created by a point positive charge. The magnitude of force acting on charges can be determined from the Coulomb's law.

$$F = F_1 + F_2 = \frac{(-q)Q}{4\pi\varepsilon_0 r_-^2} + \frac{(+q)Q}{4\pi\varepsilon_0 r_+^2}$$

Where r_- and r_+ are distances from test charge Q to charges (-q) and (+q). Putting the common factor from parentheses and using the formula of the difference of squares:

$$\frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{r_-^2} - \frac{1}{r_+^2}\right) = \frac{qQ(r_+ - r_-)(r_+ + r_-)}{4\pi\varepsilon_0 r_-^2 r_+^2}$$

Further considering that

$$|r_+ - r_-| = l, r_+ + r_- = 2r,$$

where r – average arithmetic distance from the dipole to the test charge.

$$r = \frac{r_+ + r_-}{2}$$

Than considering (42):

$$F = \frac{2Qlqr}{4\pi\varepsilon_0 r^4} = \frac{1}{4\pi\varepsilon_0}\frac{2pQ}{r^3}$$

This is force with which the dipole is drawn into the region of a stronger field. Now the explanation of the experiment with pieces of paper that are attracted to the electrified stick discussed at the beginning of this manual becomes clear.

Paper is a dielectric. Pieces of paper are polarized in the region of non-uniform field created by the stick. Therefore, they are attracted to the stick, drawing into the region of a stronger field. This is also true that in the electric field not only dielectrics are polarized, but also conductors, which will also are attracted into the region of a stronger electric field.

Armed by knowledge of the behavior of dipole in non-uniform EF, it would now be possible to explain the principle of electrophori machine, which is not as simple as it seems at first glance. So, when the disks rotate in mutually inverse directions, at some randomly taken point, it becomes formed dipole. One of the charges of this dipole is removed from the side of a positive, and the other, to a negative facing. The dipole creates a non-uniform electric field, which forces other dipoles to be drawn into it. Due to EM symmetry, those dipoles arise on different plates. The charges continue to flow to the Leiden jars, which accumulate increasing charges on them until the charge becomes so large that it causes surface currents along the dielectric, and the air breakdown, which is shown quite colorfully in demonstration experiments on physics.

CAPACITOR

It would be incomplete to discuss laws of electrostatics without paying special attention to device capable of accumulating energy in the form of electric field, so-called «capacitor».

Capacitor is a *two-terminal network, composed of two conductors separated by dielectric*. Such a device is capable of accumulating energy in the form of an electric field. The main characteristic of the capacitor is the capacitance, which is the ratio of the charge to the potential on the plates.

$$C = Q/U \qquad (45)$$

Fig. 20 shows the construction of an elementary capacitor and its schematics in electric circuit drawings. The design of capacitors is not limited to plane-parallel plates. The industry produces a variety of capacitors that differ in both the way of charge accumulation and design (Fig. 21).

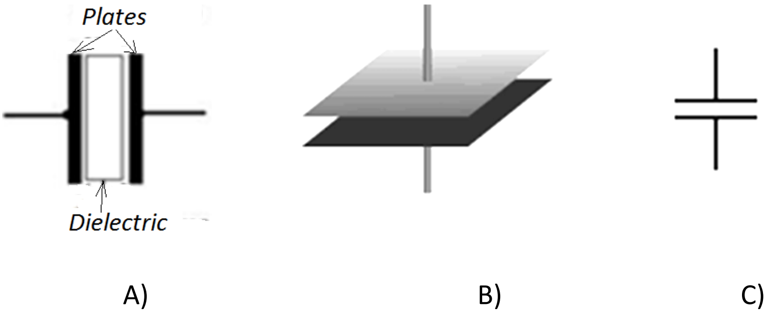


Fig. 20. Construction of flat capacitor (A, B) and its schematic in electrical circuits (C).

Fig. 21. Capacitors as devices for electric circuits.

ELECTRICAL CAPACITANCE

This is easy to determine charge stored at capacitors plates.

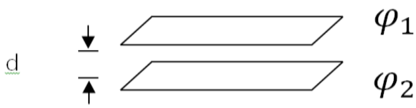


Fig. 22. To calculating charge stored at capacitor.

According to EF strength defined through potential difference:

$$E = \frac{\varphi_1 - \varphi_2}{d}; [E] = \frac{B}{M} \qquad (46)$$

Later (page 67) will be calculated EF strength, created by plane charged with charge surface density σ. For now this formula is used without derivation:

$$\frac{\partial E_y}{\partial_y}\frac{\partial E_z}{\partial_z} = \frac{\sigma}{2\varepsilon_0} \qquad (47)$$

Charge density can be defined through distance between plates, considering that since there are two plates, EF strength between them will be twice larger:

$$\sigma = \varepsilon_0 E = \varepsilon_0 \frac{\varphi_1 - \varphi_2}{d} \qquad (48)$$

Multiplying obtained result to plates area *S*, and considering

$$Q = \sigma S,$$

can be obtained charge:

$$Q = \Delta\varphi \underbrace{\frac{S\varepsilon o\varepsilon}{d}}_{C} = C(\varphi_1 - \varphi_2) \qquad (49)$$

The quantity that is a characteristic of geometric dimensions of a capacitor, and independent neither of the applied potential, nor on the field strength, is called the **capacitance** and is denoted by *C.*

$$C = \frac{\varepsilon_0 \varepsilon S}{d}, \qquad (50)$$

where ε_0 – electric constant, *S* –area of a single plate, *d* – distance between plates, ε – permittivity.

As a note: in SGS units of capacitance is *cm*. In SI capacitance is measured in farads (*F*).

$$1F = \frac{1\text{C}}{1\text{V}} = \frac{1\ \text{A}\cdot\text{s}}{1\text{V}}$$

CAPACITOR'S ENERGY

The energy of a charged capacitor can be calculated by calculating the energy of transferring the elementary charge from one plate to another. For this, force acting on the charge (*F*) should be multiplied by distance between plates (*l*).

$$A = F \cdot l = E \cdot q \cdot l = \varphi \cdot q \qquad (51)$$

$$dW = \varphi_{12} dq = \frac{q}{c} dq = \frac{qdq}{c} \qquad (52)$$

where φ_{12} – potential difference between plates. Further, by integration

$$W = \frac{1}{c} \int qdq = \frac{q^2}{2c} \qquad (53)$$

As a result, it is possible to obtain several formulas for capacitor's energy, using Eqs. (45) and (53):

$$W = \frac{1}{2} Uq = \frac{q^2}{2C} = \frac{CU^2}{2} \qquad (54)$$

ELECTRIC FIELD ENERGY DENSITY

In order to determine EF energy density, it would be the most convenient to calculate flat capacitor energy from (54) by substituting there (45) and (49),

$$W = \frac{1}{2}\left(\frac{\varepsilon\varepsilon_0 S}{d}\right)(Ed)^2 = \varepsilon\varepsilon_0 \frac{E^2}{2} S \cdot d = \varepsilon\varepsilon_0 \frac{E^2 \cdot V}{2}, \qquad (55)$$

and dividing the result by capacitors volume (V), come to energy density

$$\omega_E = \varepsilon\varepsilon_0 \frac{E^2}{2}$$

Integrating with respect to density, it is easy to calculate the energy contained in an electric field of arbitrary shape.

$$W = \int \omega_E dV = \int \varepsilon\varepsilon_0 \frac{1}{2} E^2(V) dV \qquad (56)$$

Although in engineering applications SI system is used, it is of interest to formulate the above formula in SGS system, in the form in which it is given in most textbooks on physics:

$$W = \int \frac{1}{8\pi} E^2(V) dV \qquad (57)$$

Thus, the energy density is proportional to the square of the electric field strength.

PROBLEM SOLVING EXAMPLES

Problem 1

Determine field potential from page 24 in location of charge 4.

Solution:

Potential is EF energy characteristic. It possess the additivity property, therefore the field potential at point (4) is equal to the algebraic sum of the potentials from the remaining charges. To find it, the principle of superposition can be used

$$\varphi = \varphi_1 + \varphi_2 + \varphi_3,$$

Considering that charge (1) and (3) potentials are equal and using formula: $\varphi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{kq}{r}$,

come to: $$\varphi = \frac{q}{4\pi\varepsilon_0}(2\cdot\frac{1}{a/2}+\frac{1}{r}) = \frac{q}{4\pi\varepsilon_0\cdot a}(4+\frac{1}{\cos 30^\circ});$$

$$\varphi = \frac{C\ m}{F\ m} = \frac{F/V}{F} = V$$

$$\varphi = \frac{10^{-9}}{4\pi\cdot 8{,}85\cdot 10^{-12}\cdot 0{,}2}\cdot(4+\frac{2}{\sqrt{3}}) = 232\ \text{V}$$

Considering that all charges were positive at point of interest summation was taken with plus sign.

Answer: φ = 232 V.

Problem 2

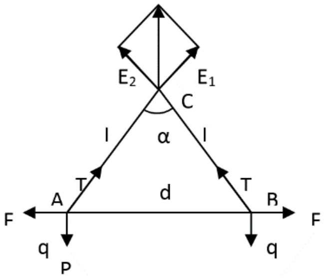
Two balls weighing 0.2g are suspended at a common point on filaments of 0.5m length. The balls were charged and the filaments moved apart at an angle of 90°. Determine the strength and potential of the field at the point of suspension of the balls.

Given:

$m = m_1 = m_2 = 0.2$ g

$l = 0.5$ m

$\alpha = 90°$



Solution:

The condition for the balance of forces in equilibrium can be written as:

$$m\vec{g} + \vec{F}_{Cl} + \vec{g} = 0$$

The equation of the balance of forces in vector form can be written as:

$$Ox: \vec{F}_{Cl} = Tsin\alpha$$

$$Oy: \vec{F}_{Cl} = Tcos\alpha$$

The distance between the balls is found by the Pythagorean theorem:

$$d = l\sqrt{2}$$

Since the balls are identical and have the same charge, only one of them can be considered. Forces acting on the ball:

$$T \sin 45^{\circ} = P$$

$$T \cos 45^{\circ} = F_{Cl}$$

Thus:

$$|F_{Cl}| = |P|$$

By substitution the formulas for the Coulomb's law and weight, taking into account the equality of charges, can be obtained:

$$k\frac{q^2}{\varepsilon d^2} = mg \Rightarrow q = \sqrt{2l^2 mg / k}$$

From the equality of charges and distances to the suspension point, it can be concluded that the electric field strengths created by these balls at the suspension point are equal in absolute value:

$$|E_{AC}| = |E_{BC}| = k\frac{q}{l^2}$$

The total strength at the point of suspension is:

$$E_C = \sqrt{E_{AC}^2 + E_{BC}^2} = \sqrt{2}|E_{AC}| = k\frac{\sqrt{2}q}{l^2} = \frac{2\sqrt{kmg}}{l}$$

$$E_C = \frac{2\sqrt{9*10^9*2*10^{-6}*9.8}}{0.5} = 1.68 kV/m$$

From the same considerations, the potentials of the electric field created by these balls at the point of suspension are also equal in absolute value: $\varphi_{AC} = \varphi_{BC} = k\frac{q}{l}$

The total potential at the point of suspension is:

$$\varphi_C = \varphi_{AC} + \varphi_{BC} = \frac{2kq}{l} = 2\sqrt{2kmg}$$

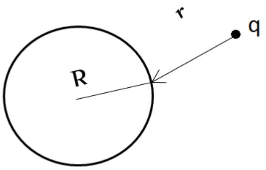
$$\varphi_C = 2\sqrt{2*9*10^9*2*10^{-6}*9.8} = 1.188 kV$$

Answer: E_C = 1.68 kV/m; φ_C = 1.188 kV.

Problem 2

The charge of 1 nC is transferred from infinity to a point 0.1 m away from the surface of a metal sphere of radius 0.1 m, charged with a surface density of 10^{-5} C/m^2. Determine the work of moving the charge.

Given:



q = 1nC = $1 \cdot 10^{-9}$ C;

r = 0.1 m;

R = 0.1;

$\sigma = 10^{-5}$ C/m^2;

Solution:

The potential difference between the initial and final points of the location of the charge is

$$\varphi_2 - \varphi_1 = \frac{A}{q}.$$

The potential of a point at infinity is zero, $\varphi_1 = 0$. φ_2 is potential at a point at a distance (R + r) from the center of the sphere. It can be found as:

$$\varphi_2 = \frac{k \cdot q_0}{(R+r)},$$

where k = $9 \cdot 10^9\ N \cdot m^2/C^2$. q_0 – charge at surface of a sphere. It can be determined knowing charge surface density.

$$\sigma = \frac{q_0}{S}, \sigma = \frac{q_0}{4 \cdot \pi \cdot R^2}, q_0 = 4 \cdot \pi \cdot R^2 \cdot \sigma.$$

Charge moving work can be determined as:

$$A = -\frac{k \cdot 4 \cdot \pi \cdot R^2 \cdot \sigma}{(R+r)} \cdot q.$$

$$|A| = 5{,}652 \cdot 10^{-5} J.$$

The sign of the work depends on the signs of charges. With the same signs, the work is performed against the forces of the field, the work is negative. With opposite signs, the work is positive (the field does the work).

Answer: $5{,}652 \cdot 10^{-5}$J.

Problem 3

Dust particle weighing $8 \cdot 10^{-15}$ kg is kept in equilibrium between horizontally arranged plates of a flat capacitor. The potential difference between the plates is 490 V, and the gap between them is 1 cm. Determine how many times the charge of a dust particle is greater than the elementary charge.

Given:

m = 8 · 10^{-15} kg;

g = 9.8 m/c^2;

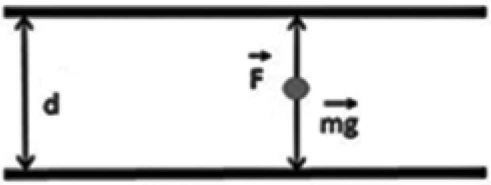
U = 490 V;

d = 1 cm = 0.01 m;

e = 1. 6 · 10^{-19} C

Find:

n-?



Solution:

In the projection on the vertical axis, the balance of forces acting on the dust particle is

$$F = mg .$$

Force acting on particle from the side of the electric field, is equalized by the gravity of the dust particle. This force acting on the side of the electric field can be easily expressed in terms of the potential on the plates:

$$F = qE = q\frac{U}{d}$$

Where q – particle's charge Thus:

$$q = \frac{mgd}{U},$$

Checking units:

$$[q] = C = \frac{kg\cdot(\frac{m}{c^2})\cdot m}{V} = \frac{kg\cdot\left(\frac{m}{c^2}\right)\cdot m\cdot C}{J} = \frac{kg\cdot\left(\frac{m}{c^2}\right)\cdot m\cdot C}{kg\cdot(\frac{m^2}{c^2})} = C;$$

The amount of excessive elementary charges can be found as

$$n = \frac{q}{e} = \frac{mgd}{Ue} = \frac{8 \cdot 10^{-15} \cdot 9,81 \cdot 0,01}{490 \cdot 1,6 \cdot 10^{-19}} = 10$$

Answer: 10.

Problem 4

Electron flies into the space between the plates of a flat capacitor in the middle of the gap in a direction parallel to the plates at a speed 2·10[7] $\frac{m}{c}$. At what minimum potential difference on the plates, electron will not fly out of the capacitor, if the length of the capacitor is 10 cm, and the distance between its plates is 1 cm?

Given:

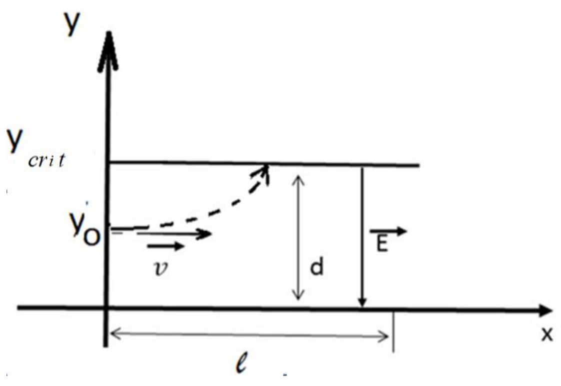
$v = 2 \cdot 10^7 \frac{m}{c}$;

$l = 10\ cm = 0.1\ m$;

$d = 1\ cm = 0.01\ m$;

$e = 1.6 \cdot 10^{-19} C$;

$m = 9.1 \cdot 10^{-31}\ kg$



Find: $U_{min.}$

Solution:

Electron in capacitor moves by the parabola. The electric field tends to attract electron to plate. At some critical value of the field, the electron will fall on the plate. This value can be calculated by making the equations of motion along the axes along and perpendicular to the capacitor plates. Considering the axis of ordinates perpendicular to the direction of flight. OY:

$$y = y_0 + v_{0y}t + \frac{at^2}{2}$$

Let y be equal to y_{crit} (critical distance between plates), and y_0 to inlet point. Than

$$y_{crit} = y_0 + \frac{at_{crit}^2}{2}, \qquad (58)$$

where t_{crit} is critical time needed for the particle to stay within capacitor while uniform motion along x axis.

Along OX axis, the motion is uniform and can be written as

$$x = x_0 + v_{0x}t$$

Choosing the coordinate of the entry point equal to zero, the critical time, can be found assuming a coordinate equal to the length of the capacitor.

$$t_{crit} = \frac{l}{v_{0x}} \qquad (59)$$

Substituting (58) to (59) can be found acceleration, which electron has to possess to stay within plates: $4 \cdot 10^{16}$ m/c^2. Now it is easy to calculate the minimum potential difference between the plates

$$ma = F = eE = e\frac{Umin}{d} => Umin = \frac{adm}{e},$$

which after calculation gives 2275 V.

Answer: $Umin = 2275\ V$.

DIVERGENCE THEOREM

Divergence theorem, also known as Ostrogradsky-Gauss theorem, (or, in short, “Gauss theorem”), which we are going to discuss, is, like the Stokes theorem, a mathematical theorem, proved by Ostrogradsky and Gauss for certain vector functions. It turned out that the divergence theorem (DT), without any modifications, establishes the connection between electric charges and the electric field created by them, representing, to some extent, a more general and elegant formulation of Coulomb's law.

Historically first DT was applied (intuitively) by Lagrange in 1867, for the purpose of transforming triple integrals into double integrals by integrating by parts.

HISTORICAL NOTE

Gauss Carl Friedrich (1777 – 1855).

Ostrogradsky Mikhail Vasilyevich (1801 – 1862).

Gauss Carl Friedrich carried out research in many branches of physics.

In 1832 created an absolute system of measures (SGS), introducing three basic units: a unit of time - 1 s, a unit of length - 1 mm, a unit of mass - 1 mg. In 1833, together with V. Weber, built the first electromagnetic telegraph in Germany. As far back as 1845, came to the idea of a finite speed of propagation of electromagnetic interactions. He studied Earth magnetism, invented a unipolar magnetometer in 1837, and in 1838 - a biophilic magnetometer. In 1829 formulated the principle of the least coercion (Gauss principle). One of the first expressed in 1818 the assumption of the possibility of the existence of non-Euclidean geometry. In 1813, 1830 showed a general method of converting the triple integral to the surface integral.

Ostrogradsky Mikhail Vasilyevich, Russian mathematician and mechanic. Studied at Kharkov University, improved knowledge in Paris.

His main scientific works were written in the field of mathematical analysis, mathematical physics, theoretical mechanics. Solved a number of important problems of hydrodynamics, theory of heat, elasticity, ballistics, electrostatics, in particular, the problem of wave propagation on the surface of a liquid (1826). Obtained a differential equation for the propagation of heat in solids and liquids. In 1826 derived the mathematical DT formulation in general form, presenting it in the form of a theorem in 1831.

ELECTRIC FIELD FLUX

In order to formulate DT, the quantity of flux has to be introduced.

EF lines **flux (Φ)** *(electric flux) is "amount" of lines intersecting with area S.* (Fig. 23). It is also called "electric flux" in short.

If all elements of the area are perpendicular to EF lines, and EF is uniform, can be written:

$$\Phi = \vec{E} \cdot \vec{S} \tag{60}$$

For a non-homogeneous field and arbitrary orientation of area with respect to EF lines, it is necessary to use the differential relation

$$d\Phi = \vec{E} \cdot d\vec{S} \tag{61}$$

Understanding by "dot" scalar multiplication.

Fig. 23. Example of EF lines flux through area of arbitrary shape.

Reminder: under **area vector** *it is understood vector normal to area and numerically equal to surface area.*

$$\vec{S} = \vec{n}S \tag{62}$$

Flux properties can be formulated as follows:

- If the total charge inside closed surface, is zero, flux through it is also zero.

-Flux is a scalar, which, depending on the direction of the field, can be either positive or negative.

As an example, consider flux through various surfaces depicted in Fig. 24. Surface A_1 is surrounded by a positive charge and flux is directed outwards, i.e. $\Phi > 0$. The surface A_2 surrounds negative charge, therefore $\Phi < 0$ and is directed inward. Assuming both charges to be equal, then flux through the surface *A* surrounding both charges is zero.

In order to formulate the DT, it is necessary to write down the EF flux through an arbitrary elementary area *dS* in the direction of the normal (Fig. 25) as

$$d\Phi_E = \vec{E}d\vec{S} = EdS\cos\alpha = E_n dS \tag{63}$$

By integrating:

$$\Phi_E = \int_S \vec{E} d\vec{S} = \int_S E_n dS \quad (64)$$

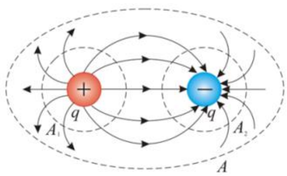


Fig. 24. Flux through different surfaces, surrounding system of charges.

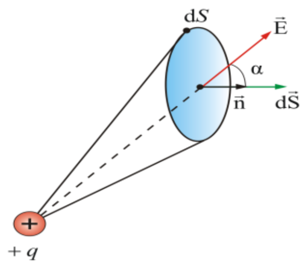


Fig. 25. To calculating flux through closed surface of arbitrary shape.

Obtained formulas can be applied to calculate the flux of the vector through the spherical closed surface *S1* surrounding the point charge.

EXAMPLE. CALCULATING FLUX THROUGH SPHERE

Let's select the center of sphere in the same place as charge (Fig. 26). Radius of sphere S_1 is R_1. At each point of the surface S_1 EF projection on the direction of the outward normal is the same and equal to

$$E_n = \frac{1}{4\pi\varepsilon_0} \frac{q}{R_1^2} \quad (65)$$

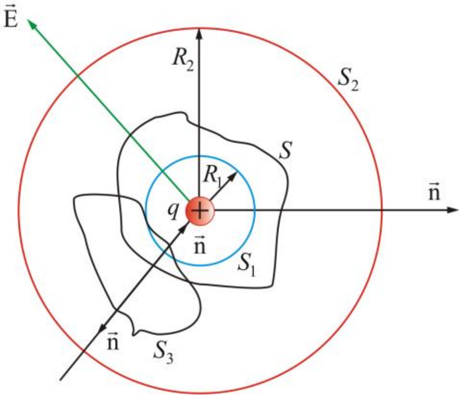


Fig. 26. To calculating flux through sphere.

Then flux though S_1 is equal to:

$$\Phi_E = \oint_{S_1} E_n \mathrm{d}S = \frac{q}{4\pi\varepsilon_0 R_1^2} 4\pi R_1^2 = \frac{q}{\varepsilon_0} \tag{66}$$

or

$$\Phi_E = \frac{q}{\varepsilon_0}. \tag{67}$$

Note that the formulas are written here in the SI system. Similarly, flux through sphere S_2 can be calculated as:

$$\Phi_E = \oint_{S_2} \frac{q}{4\pi\varepsilon_0 R_2^2} \mathrm{d}S = \frac{q}{4\pi\varepsilon_0 R_2^2} 4\pi R_2^2 = \frac{q}{\varepsilon_0} \tag{68}$$

which is the same. It can be noticed that flux does not depend on a sphere radius.

It can be proved that flux is the same for any closed surface around charges

DIVERGENCE THEOREM

It can be shown that the flux through an arbitrary closed surface *S* surrounding charge will also be equal to (67). The same is true for several charges inside the surface.

$$\Phi_E = \oint_S E_n \mathrm{d}S = \frac{\sum q}{\varepsilon_0} \tag{69}$$

From (67) also follows that flux is equal to zero through surface that does not contain charges (surface S_3, Fig. 27). Mentioned equation is called DT for the case of electric charges, although in reality the equation is more likely the consequence of Gauss-Ostrogradsky formula, the mathematical formulation of which follows below.

Equation (67) is often extended to the case of a region containing distributed charges. Since

$$\sum q_i = \int_V \rho \mathrm{d}V \tag{70}$$

and

$$\Phi_E = \oint_S \vec{E} \mathrm{d}\vec{S} = \frac{1}{\varepsilon_0} \int_V \rho \mathrm{d}V \tag{71}$$

flux can be written as

$$\Phi_E = \frac{1}{\varepsilon_0} \int_V \rho \mathrm{d}V \tag{72}$$

DIVERGENCE

To understand further material, it is necessary to explain what divergence is. Let's consider a vector function ***E*** that is continuous together with its derivatives within the closed surface and onto it, and calculate its flux through closed surface (Fig. 27),

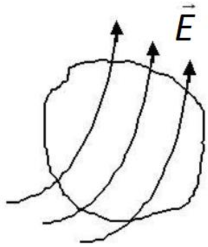


Fig. 27. Vector function flux through closed surface of arbitrary shape (planar case).

Than restrict the volume bounded by surface to zero. And calculate flux when volume approaches zero. The result of this calculation is called divergence.

$$div\vec{E} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \oint \vec{E} d\vec{S} \quad (73)$$

Equation (73) is a mathematical definition of divergence that does not depend on coordinate system. In Cartesian coordinates, divergence can be written as

$$div\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \quad (74)$$

The word "divergence" possess meaning of something flowing through. Indeed, imagining a flow of water through a certain cross-section, the divergence will be the amount of water passing through cross-section during certain time.

Similar to gradient, the divergence can be written using the operator "nabla" (38). *Nabla operator in application to scalar function has meaning of gradient, and in application to the vector one, it has meaning of divergence.*

Therefore, while *gradient is vector, divergence is scalar.*

$$div\vec{E} = \vec{\nabla} \cdot \vec{E} \tag{75}$$

DIVERGENCE THEOREM FOR ELECTRIC FIELD IN VACUUM IN DIFFERENTIAL FORM

It is convenient to use the differential formulation of DT. (62) can be applied to charge, distributed in a volume with an average density $<\rho>$. Than

$$\oint \vec{E} d\vec{S} = \frac{<\ \rho > \Delta V}{\varepsilon_o} \tag{76}$$

dividing by ΔV :

$$\frac{1}{\Delta V} \oint \vec{E} d\vec{S} = \frac{<\ \rho >}{\varepsilon_o} \tag{77}$$

Let ΔV approach zero, pulling it to the point of interest. When the volume approaches zero, average charge density becomes equal to the local density:

$$\lim_{\Delta V \to 0} \frac{<\rho>}{\varepsilon_o} = \frac{\rho}{\varepsilon_o}. \tag{78}$$

Summarizing,

$$div\vec{E} = \frac{\rho}{\varepsilon_o} \tag{79}$$

which is the formulation of DT in application to electric charges in a differential form. Writing a scalar product, the point is usually dropped in the same way as the arrow above the "nabla", Hamilton operator, therefore result looks like

$$\nabla \vec{E} = \frac{\rho}{\varepsilon_0} \quad . \tag{80}$$

DIVERGENCE THEOREM THROUGH POTENTIAL

As

(41), (75) can be re-written as:

$$div\, \vec{E} = \nabla \vec{E} = \nabla(-\nabla\varphi) = -\nabla^2\varphi \tag{81}$$

And DT can be written through potential:

$$\nabla^2\varphi = -\frac{\rho}{\varepsilon_0} \tag{82}$$

Lets introduce Laplacian Δ, which is

$$\Delta = \nabla^2 \tag{83}$$

Than Laplacian in Cartesian coordinates is

$$\Delta = \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2} \tag{84}$$

Confirming validity of (80) is suggested as homework.

GAUSS-OSTROGRADSKY FORMULA

Concluding, DT theorem can be formulated in general form, irrelevant to electric field:

$$\iiint\limits_{V} \mathrm{div}\vec{A}\mathrm{d}V = \oiint\limits_{S} \vec{A}\cdot\vec{n}\mathrm{d}S \qquad (85)$$

Here $\boldsymbol{A}$ – is any continuous vector function possessing some mathematical features related to continuity that inquisitive reader can find in specialized textbooks, such as [6].

In application to electrostatics $\boldsymbol{A}$ is replaced by electric field strength $\boldsymbol{E}$.

$$\iiint\limits_{V} \mathrm{div}\vec{E}\mathrm{d}V = \oiint\limits_{S} \vec{E}\cdot\vec{n}\mathrm{d}S \qquad (86)$$

It is not difficult to see that from the mathematical point of view (85) establishes the connection between the triple integral (in terms of volume, three variables) and double (over the surface, two variables), allowing decreasing the order of integration. Later on, in the course of electrodynamics (beyond the scope of this textbook), another theorem of this kind will be studied - the Stokes theorem, which establishes relationship between the integral over the area (two variables) to the integral along the curve (one variable), greatly simplifying the solution of many problems.

Example. Applying divergence theorem for calculating electric field created by infinite uniformly charged plane

To calculate the field strength generated by a uniformly charged infinite plane, consider a plane charged with a charge density σ.

$$\sigma = \frac{dq}{dS} \ . \qquad (87)$$

Let the plane to be cut by a cylinder, the generators of which are perpendicular to the plane, and the bases, respectively, are parallel. This is assumed that the intersection of the plane is exactly in the middle of the sides of the cylinder, Fig. 28.

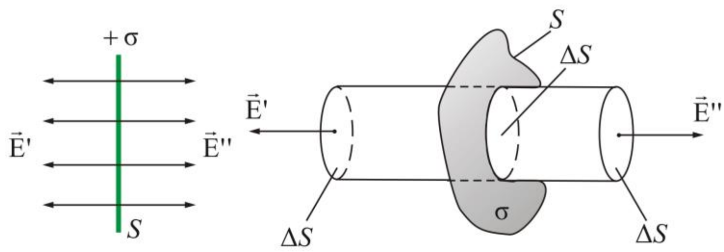


Fig. 28. To calculating electric field, created by infinite, uniformly charged plane.

Let us calculate the total flux of electric field lines Φ_E through the closed surface of the cylinder. It will be equal to the sum of the fluxes through the generators of the cylinder and the planes of its bases. By the definition of the flow (63), the flux through the generators is zero, since the field strength vector is parallel to the walls of the cylinder ($E_n = 0$). Flux through the plane of the base of the cylinder will be equal to

$$\Phi_E = 2\Delta SE \qquad (88)$$

There is an electric charge inside closed surface. From DT:

$$\Phi_E = \frac{q}{\varepsilon_0} = 2\Delta SE = \sigma\Delta S \frac{1}{\varepsilon_0} \qquad (89)$$

From which is obvious that EF strength of plane S is equal to:

$$E = \frac{\sigma}{2\varepsilon_0} \tag{90}$$

EXAMPLE. APPLYING DIVERGENCE THEOREM FOR CALCULATING STRENGTH OF THE ELECTRIC FIELD CREATED BY INFINITE, UNIFORMLY CHARGED FILAMENT

Similarly to the previous case, DT can be applied to calculate the EF intensity created by an infinite uniformly charged filament.

Filament can be modeled by a cylinder of radius *R*, charged with a constant linear density λ

$$\lambda = \frac{\mathrm{d}q}{\mathrm{d}l} \tag{91}$$

where dq – charge concentrated on a segment of the cylinder. Let's represent around the cylinder (filament) a coaxial closed surface (cylinder in the cylinder) of radius r and length l (the cylinder bases are perpendicular to the axis).

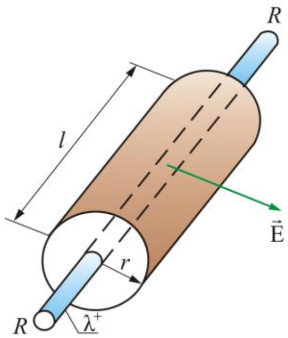


Fig. 29. To calculating EF strength, created by infinite, uniformly charged filament.

Flux through the bases of the cylinders is zero. For the lateral surface of the cylinder, flux depends on the distance r. Consequently, the flux of EF lines through the surface under consideration is equal to:

$$E_n = 0,$$

$$E_n = E(r),$$

$$\Phi_E = E(r)S = E(r)2\pi rl.$$

At

$$r \geq R,$$

Surface will be charged as

$$q = \lambda l.$$

By DT:

$$E(r)2\pi rl = \frac{\lambda l}{\varepsilon_0}$$

Than

$$E(r) = \lambda/(2\pi\varepsilon_0 r) \text{ at } r \geq R.$$

If

$$r < R,$$

than

$$E(r) = 0,$$

which can be graphically shown, taking into account vanishing of the field inside the spherical region, as follows:

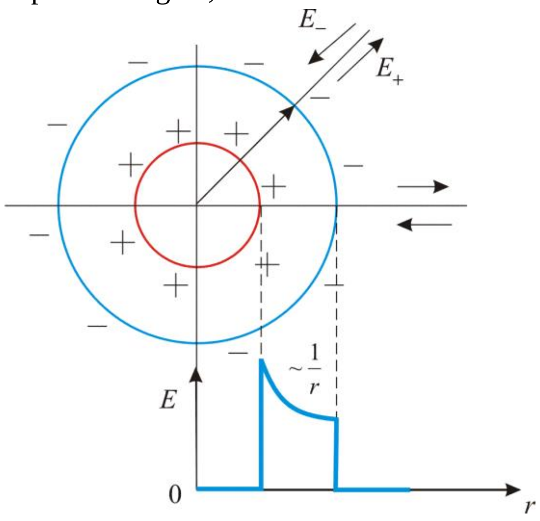


Fig. 30. Graphical dependence of EF strength on distance from uniformly charges filament of finite thickness.

Obtained result can be applied to calculate the potential at some distance from the charged filament. This can be done by integration:

$$\varphi_{21} = -\int_{P_1}^{P_2} \overline{E} \cdot d\overline{S} = -\int_{R}^{r/2} \frac{2k\lambda}{r} dr = \tag{92}$$
$$= -2\,\lambda k \ln r/2 + 2\,\lambda k \ln R$$

According to its physical meaning, the potential is defined up to a constant. This is worth noting that in the case of infinite filament, the potential at infinity cannot be taken as zero, because it would approach infinity.

It is useful to compare the dependence of the field and potential on the distance for objects of different dimensions (Table 3) and give a thought to the results obtained.

Table 3. The nature of the dependence of the field and potential on the distance from objects of different shapes / dimensions.

Object	Field	Potential	Comment
Not defined	$\sim \frac{1}{r^n}$, n≥5	$\sim \frac{1}{r^{n-1}}$	Van der Waals (dispersive) interactions
Quadrupole	$\sim \frac{1}{r^4}$	$\sim \frac{1}{r^3}$	
Dipole	$\sim \frac{1}{r^3}$	$\sim \frac{1}{r^2}$	

Point charge	$\sim\frac{1}{r^2}$	$\sim\frac{1}{r}$	$\sim\frac{1}{r}$
Filament	$\sim\frac{1}{r}$	$\sim ln\, r$	$\sim ln\, r$
Plane	Const	$\sim r$	$\sim r$

Sometimes it is stated that the field strength inside a spherically charged surface is zero due to Ostrogradsky-Gauss theorem, since there are no electric charges inside it. In fact, the DT states that only electric field divergence around a region without charges is zero, without saying anything about electric field intensity. Indeed, at some distance from point charge EF is not zero, while space may not be charged.

But, the field inside the spherical cavity actually equals zero if the surface is charged uniformly, but not by DT, but by Birkhoff 's theorem [7].

EARNSHAW'S THEOREM

Applying divergence theorem it’s easy to prove **Earnshaw’s theorem** stating that a *collection of point charges cannot be maintained in a stable stationary equilibrium configuration solely by the electrostatic interaction of the charges*. It was proven by 66Samuel Earnshaw in 1842 [[8]].

Stable equilibrium *is an equilibrium, when removed from which due to low excitation, the system returns to its original state.*

An examples of stable equilibrium are a ball bearing at the bottom and at the top spherical surface. (Fig. 31 (A) and (B).

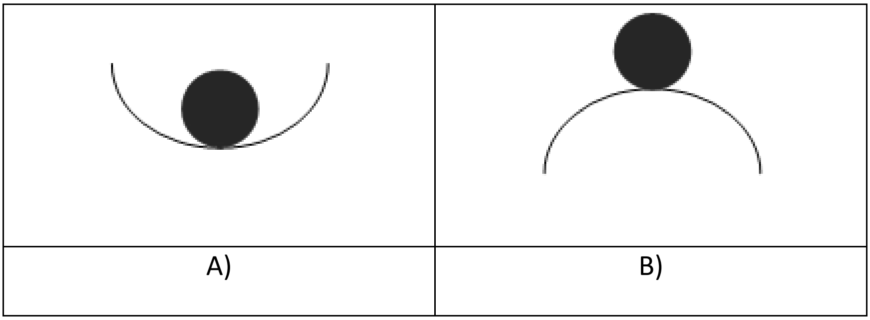


Fig. 31. Examples of stable (A) and unstable equilibrium (B).

Another example of equilibrium is a ball bearing at the top of the saddle point (Fig. 32). In the XY plane, the position is stable, and in the XZ plane, unstable. If at least in one dimension it is unstable, then such equilibrium is unstable.

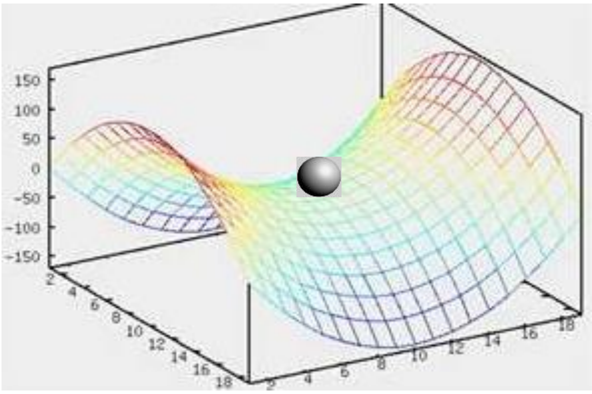


Fig. 32. Unstable equilibrium at a saddle point.

Theorem can be proven with the help of divergence theorem by contradiction method.

Method "by contradiction" *is based on the assumption that the asserted position is false.* It shows that such a statement contradicts the true assumption. (Either proven previously or taken without evidence.)

To prove it by contradiction, lets surround the test charge with a sphere in which there are no other charges. Suppose that outside this sphere there are other charges, but the test charge is in equilibrium, that is, the sum of the forces acting on it is equal to zero.

If the test charge within the sphere is in equilibrium, than ***E*** created by charges outside the sphere acting on it is equal to zero. Under small fluctuation there should be created force returning test charge in equilibrium position. This force can be understood as force arising from the charge of an opposite sign appearing symmetrically to the point of equilibrium. But appearing another charge within the sphere would change flux through the sphere which contradicts to divergence theorem. Therefore, equilibrium is unstable and the theorem is proven.

Conclusion

From this book reader learned how to calculate potential and strength of electric field, created by a system of charges or conductors of various shapes. Basic ideas about the energy of electric field, and properties of electric charges were set. Theory how works such important device as capacitor was discussed in direct current circuits. Necessary mathematical background of vector analysis given to make reader handy with such quantities as divergence, gradient, Laplace operator in application to electric field. This is believed that obtained knowledge will assist reader in further studying the course of electricity and magnetism and deeper understanding of electrostatic phenomena's.

Problems

Coulomb's law

Problem 1

Charges of 1 nC are placed at the vertices of an equilateral triangle with a side of 0.2 m. The equatorial force acting on the fourth charge placed on the middle of one side of the triangle is equal to 0.6 μN. Determine this charge, the strength and potential of the field at its location.

Problem 2

If the negative charge is placed in the center of the square, at the vertices of which there are charges of +2 nC, the resultant force acting on each charge is zero. Calculate the numerical value of the negative charge.

Potential. Work of electric field

Problem 3

Two identical charges are in the air at a distance of 0.1 m from each other. The field strength at a point removed at a distance of 0.06 m from one and 0.08 m from another charge is equal to 10 kV / m. Determine the potential of the field at this point and the value of the charges.

Problem 4

The charge of - 1nC has moved in the field created by charge +1.5 nC from the point with a potential of 100V to the point with a potential of 600 V. Determine the work of the field forces and the distance between these points.

Problem 5

Two identical charges are in the air at a distance of 0.1 m from each other. The field strength at a point removed at a distance of 0.06 m from one and 0.08 m from another charge is equal to 10 kV / m. Determine the potential at this point and the value of the charges.

Problem 6

Charges of 1 nC are placed at the vertices of an equilateral triangle with a side of 0.2 m. The equatorial force acting on the fourth charge, placed on the middle of one side of the triangle, is 0.6 μN. Determine this charge, the strength and potential of the field at its location.

Problem 7

The charge of 1 nC was attracted to an infinite plane, uniformly charged with a surface density of 0.2 μC / m2. At what distance from the plane was the charge, if the work of the field forces on its displacement was equal to 1 μJ?

Problem 8

The charge of 1 nC is transferred from infinity to a point 0.1 m away from the surface of a metal sphere of radius 0.1 m, charged with a surface density of 10^{-3} C/m^2. Determine the work to move the charge.

Problem 9

Two balls weighing 2 mg are suspended at a common point on filaments 0.5 m long. The balls were charged and the filaments dispersed at an angle of 90°. Determine the strength and potential of the field at the point of suspension of the balls.

Problem 10

Determine work to move charges of 1 and 2 nC, initially at 0.5 m away from each other, to the distance 0.1 m from each other?

Electric field flux. Divergence theorem

Problem 11

The surface charge density of an infinite uniformly charged plane is 30 nC/m^2. Determine the flux of the electric field through the surface of a sphere with a diameter of 15 cm, cut by this plane in half.

Problem 12

In the field of an infinite uniformly charged plane with a surface charge density of 10 μC/m^2, a charge moves from a point 0.1 m away from the plane to a point 0.5 m away from it. Determine the charge, if the work is done at 1 mJ.

Capacitor. Electric field energy

Problem 13

Calculate the capacity of a battery consisting of three 1 μF capacitors each, connected in all possible ways.

Problem 14

The energy of a flat air capacitor is 0.4 nJ, the potential difference between plates is 600 V, the area of the plates is 1 cm^2. Determine the distance between the plates, the strength and the volume density of the field energy of the capacitor.

Problem 15

The flat air condenser is charged to a potential difference of 300 V. The area of the plates is 1 cm^2, the field strength in the gap between them is 300 kV/m. Determine the surface charge density on the plates, capacitance and energy of the capacitor.

Problem 16

Electric field is created by infinite, uniformly charged plane with a surface charge density of 10 μC / m^2. Charge moves from a point 0.1 m away from the plane to a point 0.5 m away from it. Determine the charge, if the work done is 1 mJ.

Problem 17

Two capacitors of the same capacity of 3 μF are charged one to 100 V and the other to 200 V. Determine the voltage between the capacitor plates if they are connected in parallel: a) plates of the same charge b) plates of the opposite charge.

Problem 18

The charge on each of the two series capacitors with capacitances of 18 and 10 pF is equal to 0.09 nC. Determine the voltage: a) on the battery of capacitors; b) on each capacitor.

Problem 19

Under the action of an attractive force of 1 mN, the dielectric between the capacitor plates is at a pressure of 1 Pa. Determine the energy and the volume density of the energy of the capacitor field, if the distance between its plates is 1 mm.

Problem 20

A capacitor with a capacity of 6 μF is connected in series with a capacitor of unknown capacitance and they are connected to a DC 12 V source. Determine the capacitance of the second capacitor and the voltage on each capacitor if the charge of the system of the capacitors is 24 μC.

Problem 21

Find the volumetric energy density of the electric field created by a charged metal sphere with a radius of 5 cm at a distance of 5 cm from its surface, if the surface density of charge on it is 2 μC / m^2.

Problem 22

The charge of 1nC is 0.2 m from an infinitely long uniformly charged filament. Under the action of the filament field, the charge moves by 0.1 m. Determine the linear density of the filament charge, if the work of the field forces is 0.1 μJ.

Problem 23

A capacitor with a paraffin dielectric is charged to a potential difference of 150 V. The field strength is 6 MV / m, the plate area is 6 cm^2. Determine the capacitance of the capacitor and the surface charge density on the capacitor plates.

Problem 24

The plate area of a flat mica capacitor is 1.1 cm^2, the gap between them is 3 mm. When the capacitor was discharged, energy of 1 μJ was released. To what potential difference was the capacitor charged?

Table of physical constants

Quantity	Symbol	Value
Electric constant (vacuum permittivity)	ε_0	$8{,}85418781762039 \cdot 10^{-12}$ F/m
Coulomb's constant	k	$8.987\ 742\ 438 \cdot 10^{9}$ $N \cdot m^2 C^{-2}$
Electron charge (elementary charge)	e^-	$-1{,}6021766208 \cdot 10^{-31}$ C
Electron mass	m_e	$9{,}10938291 \cdot 10^{-31}$ kg
Electromagnetic constant (speed of light)	c	299 792 458 m/s
Proton mass	m_p	$1{,}672\ 621\ 898 \cdot 10^{-27}$ kg
Planck's constant	h	$6{,}626\ 070\ 040 \cdot 10^{-34}$ J·s

Acronyms

EF	Electric field
EP	Electric potential
EqP	Equipotential
LCEC	Law of conservation of electric charge
DT	Divergence theorem

Alphabetical Index

References

1. Millikan R. A. (1913). "On the elementary electrical charge and the Avogadro constant" (PDF). Physical Review. Series II. 2: 109–143.

2. Einstein A. (1916), Relativity: The Special and General Theory (Translation 1920), New York: H. Holt and Company.

3. L.D. Landau, E.M. Lifshitz and L.P. Pitaevskii, Electodynamics of continuous media, Vol. 8., Translated from the second edition of Elektrodinamika sploshnykh sred, Izdatel'stvo "Nauka", Moscow . Elsevier, reprinted in.2004.

4. P. Debye , Physik. Z., 36, 193, 1935.

5. P. Debye , Chem. Rev., 19, 171, 1936.

6. G.M. Fichtenholz, Fundamentals of Mathematical Analysis, any edition.

7. Birkhoff, G. D. (1923). Relativity and Modern Physics. Cambridge, Massachusetts: Harvard University Press. LCCN 23008297.

8. Earnshaw, Samuel (1842). "On the Nature of the Molecular Forces which Regulate the Constitution of the Luminiferous Ether". Trans. Camb. Phil. Soc. 7: 97–112.

FOR NOTES